第六届湖北省高三（4 月）调研模拟考试数学参考答案

一 选择题

1	2	3	4	5	6	7	8	9	10	11	12
B	A	B	D	C	C	D	C	ABC	CD	ABD	AC

二、填空题

13. $2\sqrt{2}$ 14. 11π 15. 3 16. m<-2

三、解答题

17 设{a_n}公比 q,{b_n}公比 d,则：

$$\begin{cases}2b_2=b_4-1\\b_5=3b_2\end{cases}\Rightarrow\begin{cases}2b_1+2d=b_1+3d-1\\b_1+4d=3b_1+3d\end{cases}\Rightarrow\begin{cases}b_1=1\\d=2\end{cases}\Rightarrow b_n=2n-1$$

$\therefore a_2=b_5=9$

若选① $a_3=b_4+20=27\Rightarrow q=\dfrac{a_3}{a_2}=3,\Rightarrow a_n=3^n$

若选② $a_1=b_2=3\Rightarrow q=\dfrac{a_2}{a_1}=3,\Rightarrow a_n=3^n$

若选③ $S_3=39\Rightarrow a_2(\dfrac{1}{q}+1+q)=39,\Rightarrow q=3,\dfrac{1}{3}$ 又因为{a_n}递增，

$\therefore q=3\Rightarrow a_n=3^n.$ $\dfrac{1}{b_n\log_3 a_{2n+1}}=\dfrac{1}{(2n-1)(2n+1)}=\dfrac{1}{2}(\dfrac{1}{2n-1}-\dfrac{1}{2n+1})$

$$T_n=\frac{1}{2}(\frac{1}{1}-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}+\cdots+\frac{1}{2n-1}-\frac{1}{2n+1})=\frac{n}{2n+1}$$

18(1)

$$f(x)=\sin x(\frac{\cos x}{2}-\frac{\sqrt{3}\sin x}{2})=\frac{\sin x\cos x}{2}-\frac{\sqrt{3}\sin^2 x}{2}$$

$$=\frac{\sin 2x}{4}+\frac{\sqrt{3}\cos 2x}{4}-\frac{\sqrt{3}}{4}=\frac{\sin(2x+\frac{\pi}{3})}{2}-\frac{\sqrt{3}}{4}$$

令 $2k\pi-\dfrac{\pi}{2}\le 2x+\dfrac{\pi}{3}\le 2k\pi+\dfrac{\pi}{2},k\in Z\Rightarrow k\pi-\dfrac{5\pi}{12}\le x\le k\pi+\dfrac{\pi}{12}$

f(x)的单调增区间是 $[k\pi-\dfrac{5\pi}{12}，k\pi+\dfrac{\pi}{12}],k\in Z$

(2) $\dfrac{\sin(2A+\frac{\pi}{3})}{2}-\dfrac{\sqrt{3}}{4}=-\dfrac{\sqrt{3}}{4}\Rightarrow 2A+\dfrac{\pi}{3}=k\pi,k\in Z,\because$ A 为锐角，$\therefore A=\dfrac{\pi}{3}$

由余弦定理得：$a^2=b^2+c^2-2bc\cos A\Rightarrow b^2+c^2-bc=5\Rightarrow(b+c)^2-3bc=5$

又$b+c=3 \Rightarrow bc=\frac{4}{3}$

面积$S=\frac{1}{2}bc\sin A=\frac{1}{2}\frac{4}{3}\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{3}$

19（1）连B_1N 由 $B_1P // BC // AD, B_1P \not\subset 平面AMD, \therefore B_1P // 平面AMD$

$AN=B_1M, AN // B_1M$

四边形 ANB_1M 为平行四边形,

$\therefore NB_1 // AM \quad \because B_1N \not\subset 平面AMD, \therefore B_1N // 平面AMD$

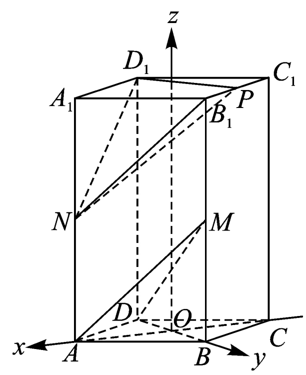
$\because NB_1 \cap B_1P=B_1, \therefore 平面NB_1P // 平面AMD. \because PN \subset 平面NB_1P,$

$\therefore PN // 平面AMD$

（2）连 AC 交 BD 于 O，有 $AC \perp BD,$ 以 O 为原点，OA、OB 为 x 轴、y 轴正方向，建立空间直角坐标系

$A(\sqrt{3},0,0), D(0,-1,0), M(0,1,2), P(-\frac{\sqrt{3}}{2},\frac{1}{2},4), N(\sqrt{3},0,2), D_1(0,-1,4), \vec{DA}=(\sqrt{3},1,0), \vec{DM}=(0,2,2),$

$\vec{PN}=(\frac{3\sqrt{3}}{2},-\frac{1}{2},-2), \vec{PD}=(\frac{\sqrt{3}}{2},-\frac{3}{2},0),$



设平面 AMD 法向量$\vec{\alpha}=(x,y,z)$

$$\begin{cases}\vec{\alpha}\cdot\vec{DA}=0\\ \vec{\alpha}\cdot\vec{DM}=0\end{cases} \Rightarrow \begin{cases}\sqrt{3}x+y=0\\ 2z+2y=0\end{cases}, 取\vec{\alpha}=(1,-\sqrt{3},\sqrt{3})$$

设平面 PND_1 法向量 $\vec{\beta}=(x',y',z')$

$$\begin{cases}\vec{\beta}\cdot\vec{PN}=0\\ \vec{\beta}\cdot\vec{PD}=0\end{cases} \Rightarrow \begin{cases}\frac{3\sqrt{3}}{2}x'-\frac{1}{2}y'-2z'=0\\ \frac{\sqrt{3}}{2}x'-\frac{3}{2}y'=0\end{cases}, 取\vec{\beta}=(\sqrt{3},1,2)$$

平面 AMD 与平面 PND_1 所成锐二面角余弦值为

$$\left|\cos<\vec{\alpha},\vec{\beta}>\right|=\frac{|\vec{\alpha}\cdot\vec{\beta}|}{|\vec{\alpha}|\cdot|\vec{\beta}|}=\frac{\sqrt{42}}{14}$$

20（1）得分 80 以上的人数为$100\times10\times(0.008+0.002)=10$，X 可能取值为 0，1，2

$P(X=0)=\frac{C_{90}^{2}}{C_{100}^{2}}=\frac{89}{110},P(X=1)=\frac{C_{10}^{1}C_{90}^{1}}{C_{100}^{2}}=\frac{2}{11},P(X=2)=\frac{C_{10}^{2}}{C_{100}^{2}}=\frac{1}{110}$，X 分布列为

X	0	1	2
P	$\frac{89}{110}$	$\frac{2}{11}$	$\frac{1}{110}$

$E(X)=0\times\frac{89}{110}+1\times\frac{2}{11}+2\times\frac{1}{110}=\frac{1}{5}$

(2)

$\bar{x}=10\times(35\times0.002+45\times0.009+55\times0.022+65\times0.033+75\times0.024+85\times0.008+95\times0.002)=65$

$s^{2}=(35-65)^{2}\times10\times0.002+(45-65)^{2}\times10\times0.009+(55-65)^{2}\times10\times0.022+(75-65)^{2}\times10\times0.024$
$+(85-65)^{2}\times10\times0.008+(95-65)^{2}\times10\times0.002=150$

取$\mu=\bar{x}=65$，$\sigma=\sqrt{s^{2}}=12.2$

① $P(77.2<Y<89.4)=\frac{1}{2}[P(\mu-2\sigma<Y<\mu+2\sigma)-P(\mu-\sigma<Y<\mu+\sigma)]=0.1359$

② $Z\sim B(2000,0.1359),E(Z)=2000\times0.1359=271.8$

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(1)依题意得：

$|AF_1|=2,|AF_2|=4,|F_1F_2|=2\sqrt{3}.\therefore 2a=|AF_2|-|AF_1|=2,a=1$

$2c=|F_1F_2|=2\sqrt{3},c=\sqrt{3}$，$b^{2}=c^{2}-a^{2}=2$

此时Γ的方程为$x^{2}-\frac{y^{2}}{2}=1$

(2)设 l 的方程为$x=my-c,$与$\frac{x^{2}}{a^{2}}-\frac{y^{2}}{b^{2}}=1$ 联立，得 $(b^{2}m^{2}-a^{2})y^{2}-2b^{2}cmy+b^{4}=0$

设$A(x_1,y_1)$ $B(x_2,y_2)$，则$y_1+y_2=\frac{2b^{2}cm}{b^{2}m^{2}-a^{2}}$ $y_1y_2=\frac{b^{4}}{b^{2}m^{2}-a^{2}}$，由$AF_2\perp BF_2$

$\overrightarrow{F_2A}\cdot\overrightarrow{F_2B}=0$ $(x_1-c)(x_2-c)+y_1y_2=0$

$(my_1-2c)(my_2-2c)+y_1y_2=0\Rightarrow(m^{2}+1)b^{4}-4m^{2}c^{2}b^{2}+4c^{2}(b^{2}m^{2}-a^{2})=0$

$\Rightarrow(m^{2}+1)b^{4}=4a^{2}c^{2}\Rightarrow(m^{2}+1)=\frac{4a^{2}c^{2}}{b^{4}}\geq1\Rightarrow4a^{2}c^{2}\geq(c^{2}-a^{2})^{2}$

$\therefore c^{4}+a^{4}-6a^{2}c^{2}\leq0\Rightarrow e^{4}-6e^{2}+1\leq0,$又$\because e>1\therefore1<e^{2}\leq3+2\sqrt{2}$

$\therefore1<e\leq1+\sqrt{2}$

又 A、B 在左支且l过 F_1，

$\therefore y_1y_2<0,\frac{b^{4}}{b^{2}m^{2}-a^{2}}<0\Rightarrow m^{2}<\frac{a^{2}}{b^{2}}\Rightarrow m^{2}+1=\frac{4a^{2}c^{2}}{b^{4}}<\frac{a^{2}}{b^{2}}+1$

$\therefore4a^{2}<b^{2}=c^{2}-a^{2}\Rightarrow e^{2}>5$

综上所述 $\sqrt{5} < e \le 1+\sqrt{2}$

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(1) $f'(x) = 2ae^{2x} + (2a-1)e^x - 1 = (e^x+1)(2ae^x - 1)$

$a \le 0$时，$f'(x) < 0, f(x)$在$(-\infty,+\infty)$上递减；

$a > 0$时，$f'(x) = 0, x = \ln\dfrac{1}{2a}$

$x < \ln\dfrac{1}{2a}, f'(x) < 0, f(x)$递减；

$x > \ln\dfrac{1}{2a}, f'(x) > 0, f(x)$递增；

综上所述，

$a \le 0$时，$f'(x) < 0, f(x)$在$(-\infty,+\infty)$上递减；

$a > 0$时，$f(x)$在$(-\infty, \ln\dfrac{1}{2a})$递减；$f(x)$在$(\ln\dfrac{1}{2a},+\infty)$递增

(2)

令$g(x) = f(x) - (3a-1)\cos x = ae^{2x} + (2a-1)e^x - x - (3a-1)\cos x$

当$\mathrm{x} \ge 0, \mathrm{g(x)} \ge 0,$取$x = \dfrac{\pi}{2}, \Rightarrow a \ge \dfrac{e^{\frac{\pi}{2}} + \frac{\pi}{2}}{e^{\pi} + 2e^{\frac{\pi}{2}}} > 0 \Rightarrow a > 0$

$g'(x) = 2ae^{2x} + (2a-1)e^x - 1 + (3a-1)\sin x$

当$a \ge \dfrac{1}{2}$时，由（1），$\ln\dfrac{1}{2a} \le 0, f(x)$在$[0,+\infty)$上单增；

$g(x) \ge f(0) - (3a-1)\cos x = (3a-1)(1-\cos x) \ge 0$ 满足题意

当$0 < a \le \dfrac{1}{3}$时,$\because -1 \le \sin x \le 1, g'(x) \le 2ae^{2x} + (2a-1)e^x - 1 - (3a-1) = 2ae^{2x} + (2a-1)e^x - 3a$

令$e^x = t, h_1(t) = 2at^2 + (2a-1)t - 3a, h_1(1) = a - 1 < 0,$由$h_1(t)$是开口向上的二次函数，

故存在$x_1 < 1 < x_2,$使得$\mathrm{h}_1(x_1) = h_1(x_2) = 0,$且$1 \le t < x_2$时，$\mathrm{h}_1(t) < 0$

$\therefore 0 \le x < \ln x_2$时，$g'(x) < 0, g(x)$单减$\Rightarrow \mathrm{g(x)} < \mathrm{g(0)} = 0,$不合题意；

当$\dfrac{1}{3} < a < \dfrac{1}{2}$时，令$\mathrm{F}(x) = e^x - 1 - \sin x \quad (x \ge 0), \mathrm{F}'(x) = e^x - \cos x \ge 1 - \cos x \ge 0 \Rightarrow F(x)$单增，

$\therefore \mathrm{F}(x) \ge F(0) = 0,$即$\sin x \le e^x - 1$

$\therefore \mathrm{g}'(x) \le 2ae^{2x} + (2a-1)e^x - 1 + (3a-1)(e^x - 1) = 2ae^{2x} + (5a-2)e^x - 3a$

令$\mathrm{e}^x = t, h_2(t) = 2at^2 + (5a-2)t - 3a, h_2(1) = 4a - 2 < 0,$

同理可得，故存在$x_1' < 1 < x_2',$使得$\mathrm{h}_2(x_1') = h_2(x_2') = 0,$且$1 \le t < x_2'$时，$\mathrm{h}_2(t) < 0$

$\therefore 0 \le x < \ln x_2'$时，$g'(x) < 0, g(x)$单减$\Rightarrow \mathrm{g(x)} < \mathrm{g(0)} = 0,$不合题意；

综上所述：$a \ge \dfrac{1}{2}$